Moving Clocks Do Not Always *Appear* to Slow Down: Don't Neglect the Doppler Effect

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In popular accounts of the time dilation effect in Einstein's special relativity, one often encounters the statement that moving clocks run slow. For instance, in the acclaimed PBS program "NOVA,"[1] Professor Brian Greene says, "[I]f I walk toward that guy … he'll perceive my watch ticking slower." Also in his earlier piece for *The New York Times*,[2] he writes that "if from your perspective someone is moving, you will see time elapsing slower for him than it does for you. Everything he does … will appear in slow motion." We need to be careful with this kind of description, because sometimes authors neglect to consider the finite time of signal exchange between the two individuals when they *observe* each other. This article points out that when two individuals approach each other, everything will actually appear in fast motion—a manifestation of the relativistic Doppler effect.[3]

Numerous publications use thought experiments to demonstrate that it takes a longer time for a clock in motion to tick as compared with our own clock at rest (often using a clock consisting of two mirrors and a pulse of light bouncing between them).[4] But a less emphasized fact is that we cannot see the moving clock instantaneously because of the finite speed of light. To understand how the ticking of the moving clock appears to an observer at rest, it might help with the analogy of the classical Doppler effect, which is due to the finite speed of sound.[5] Most people have the experience of hearing the sudden drop in frequency of the siren of a passing ambulance. In relativity, the qualitative effect is the same for the pitch of a siren and the frequency of a spectral line, say, the cesium-133 atom, which is used to define the SI unit second. A person will thus appear in fast (slow) motion if he/she approaches (recedes from) the observer. The quantitative effect is discussed in standard textbooks; here we use the graphical representation similar to an earlier article published in this journal to analyze the phenomenon.[6]

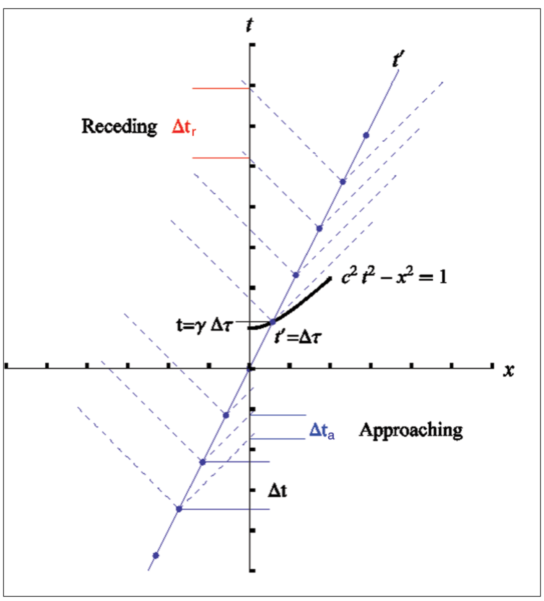

Fig. 1. The time interval between two consecutive light signals Δt_a (Δt_r) received by a lab observer is not equal to $\Delta t = \gamma\Delta\tau$, but is less (greater) than the time interval between two consecutive ticks of the lab's clock as the rocket is approaching (receding).

In the spacetime diagram (*xt* plane) of an observer in the lab shown in Fig. 1, the line marked t' represents the world line of the origin of the rocket moving at the constant speed v relative to the lab. The hyperbola $(ct)^2 - x^2 = 1$, where c is the speed of light, is an invariant, and its intersections with the t and t' axes set the time scales in the lab and rocket frames, respectively. The reader might consult the classic *Spacetime Physics* for a worked example.[7] We use $\Delta\tau$ for the time scale indicated by the rocket's clock. As shown in the figure, the corresponding time interval in the lab Δt is greater by a factor of $\gamma = 1/\sqrt{(1 - v^2/c^2)}$. One *deduces* that the rocket's clock slows down, but this is not what it will *appear*. The 45° dashed lines in the figure represent light signals emitted at each tick of the rocket's clock, and it is apparent that the observer in the lab's origin (t-axis) receives light signals more frequently when the rocket is approaching.

To find the time interval Δt_a between consecutive signals received by the lab observer, we should consider both time dilation and the change of distance as the rocket is approaching. Because the time for the next light signal to arrive at the lab's origin is reduced by the distance the rocket traveled $v\Delta t$ divided by the speed of light c, we obtain

$$\Delta t_a = \Delta t - \frac{v\Delta t}{c} = \left(\frac{1-\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}\right)\Delta\tau = \sqrt{\frac{c-v}{c+v}}\Delta\tau. \tag{1}$$

Our calculation reveals that Δt_a is invariably less than $\Delta\tau$, thus we will see the approaching clock tick faster. Notice that if we let c after the first equal sign be infinity, meaning that the signal is transmitted instantaneously, $\Delta t_a = \Delta t = \gamma\Delta\tau > \Delta\tau$; this is the oversight committed by some authors when stating that a moving clock runs slow without regarding the direction of motion.

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Similarly, after the rocket passes the lab and recedes from it, the distance that the next signal needed to travel to the lab is increased by $v\Delta t$, and the time interval Δt_r of consecutive signals received by the lab observer is

$$\Delta t_r = \Delta t + \frac{v\Delta t}{c} = \left(\frac{1+\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} \right) \Delta\tau = \sqrt{\frac{c+v}{c-v}}\Delta\tau. \qquad (2)$$

Because Δt_r is invariably greater than $\Delta\tau$, we will indeed see the clock receding from us run slow, but slower than merely considering the time dilation effect.

We leave an exercise for students: set $\Delta\tau$ to unity and graph Δt, Δt_a, and Δt_r as a function of v/c. It should be evident that the observed time scales (Δt_a and Δt_r) and the deduced one from the thought experiment (Δt) are significantly different. So next time when we encounter the clock problem in special relativity, we must be aware that the observed time is the combined effects of time dilation and motion of the clock.

References

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